Formal Synthesis of Stabilizing Controllers for Switched Systems

Pavithra Prabhakar & Miriam García Soto
Kansas State University & IMDEA Software Institute

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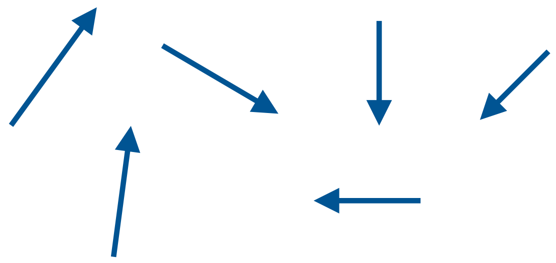
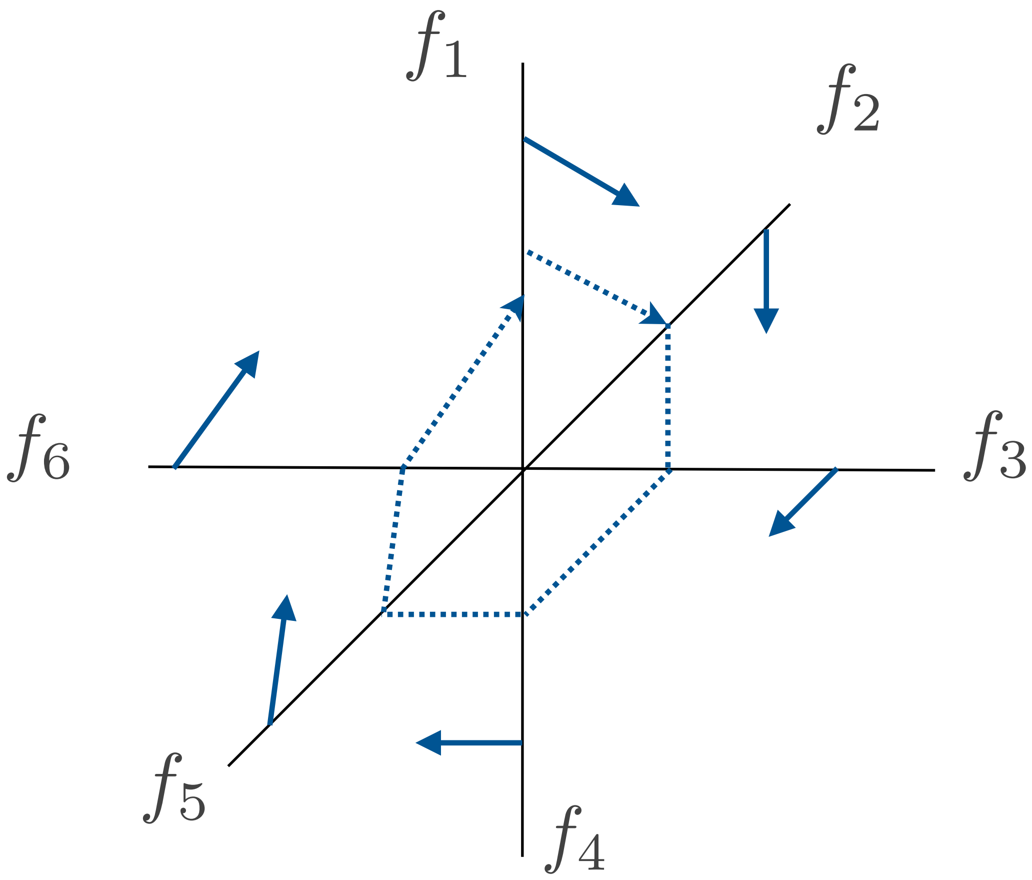
Switching logic synthesis

Dynamics



Partition



Given a set of dynamics and a partition, assign dynamics to each facet such that the resulting switched system is stable.

Switched system

Family of dynamical systems

$$\mathcal{S} = (\mathcal{P}, \{g_p\}_{p\in\mathcal{P}})$$

$$\dot{x}(t) \in g_p(x(t)), \quad p \in \mathcal{P}$$

$$g_p : \mathbb{R}^n \to 2^{\mathbb{R}^n}$$

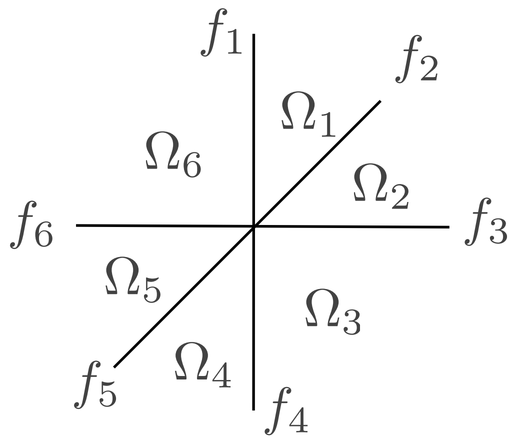
Switching strategy

$$\alpha : \mathcal{F}^+ \to \mathcal{P}$$
$$f_i, f_j, \ldots, f_l \mapsto p$$

Partition - finite set of valid facets

$\mathcal{R} = \{\Omega_1, \Omega_2, \ldots, \Omega_k\}$ closed convex polyhedra

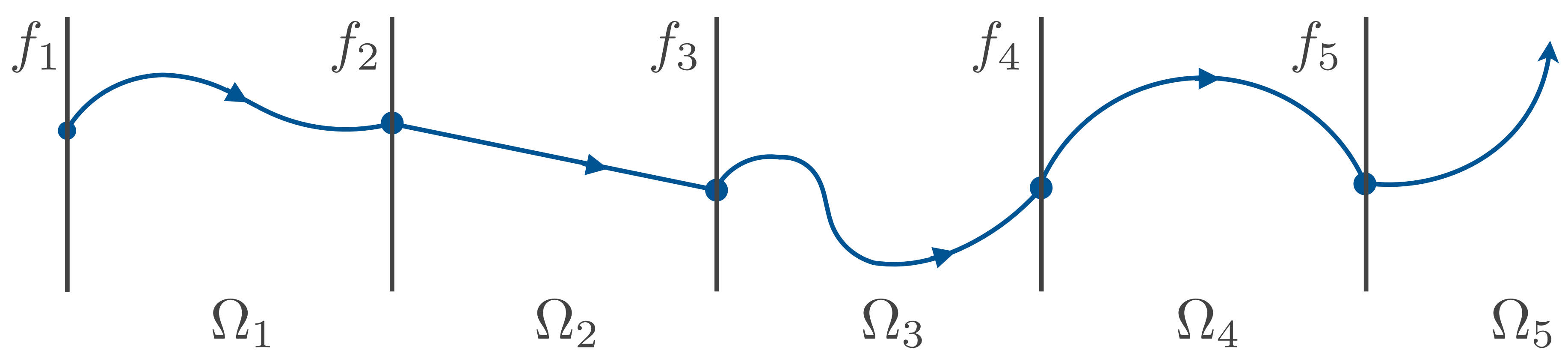
- ☐ $\mathbb{R}^n = \cup_{\Omega\in\mathcal{R}}\Omega$
- ☐ $\mathring{\Omega}_i \neq \emptyset$ for every i
- ☐ $\mathring{\Omega}_i \cap \mathring{\Omega}_j = \emptyset$ for every $i \neq j$



$\mathcal{F} = \{f_1, f_2, \ldots, f_k\}$

maximal closed convex subsets of boundary of Ω's

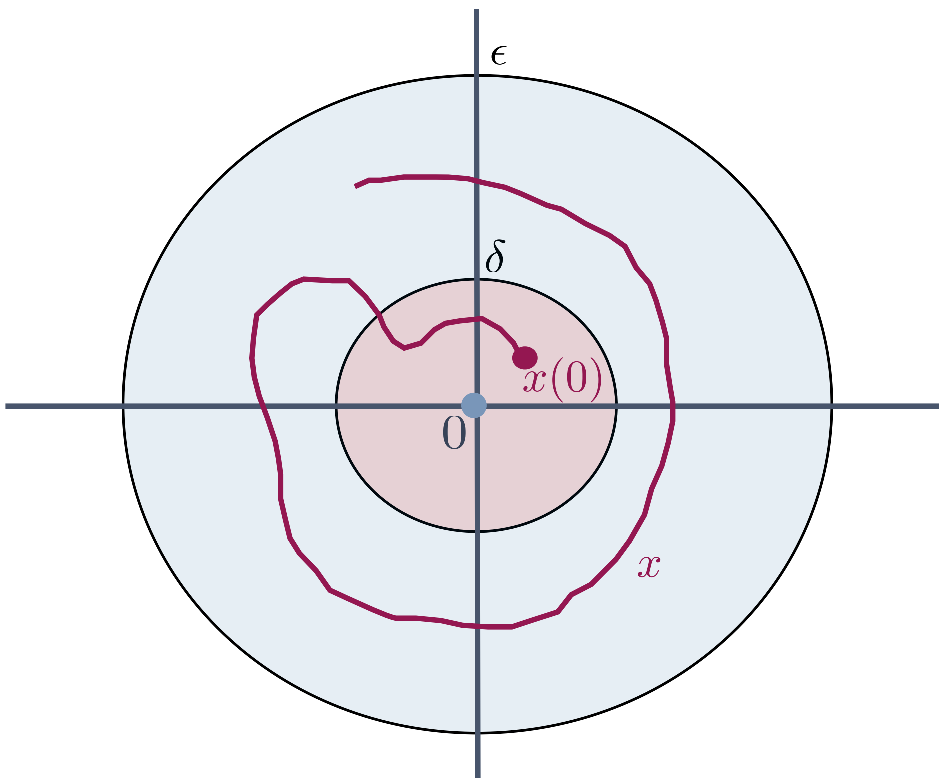
Switched system $\mathcal{S}_\alpha = (\mathcal{P}, \{g_p\}_{p\in\mathcal{P}}, \alpha)$

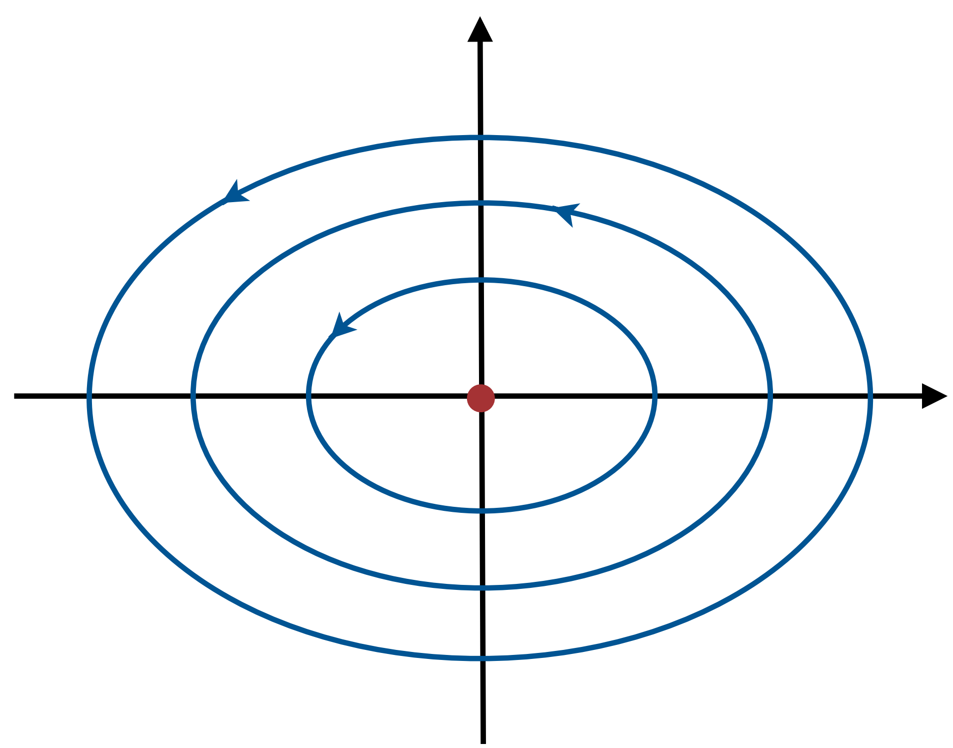


Stabilization problem

Given a system $\mathcal{S}$ and a set of valid facets $\mathcal{F}$, find a switching strategy $\alpha : \mathcal{F}^+ \to \mathcal{P}$, such that the switched system $\mathcal{S}_\alpha$ is stable.

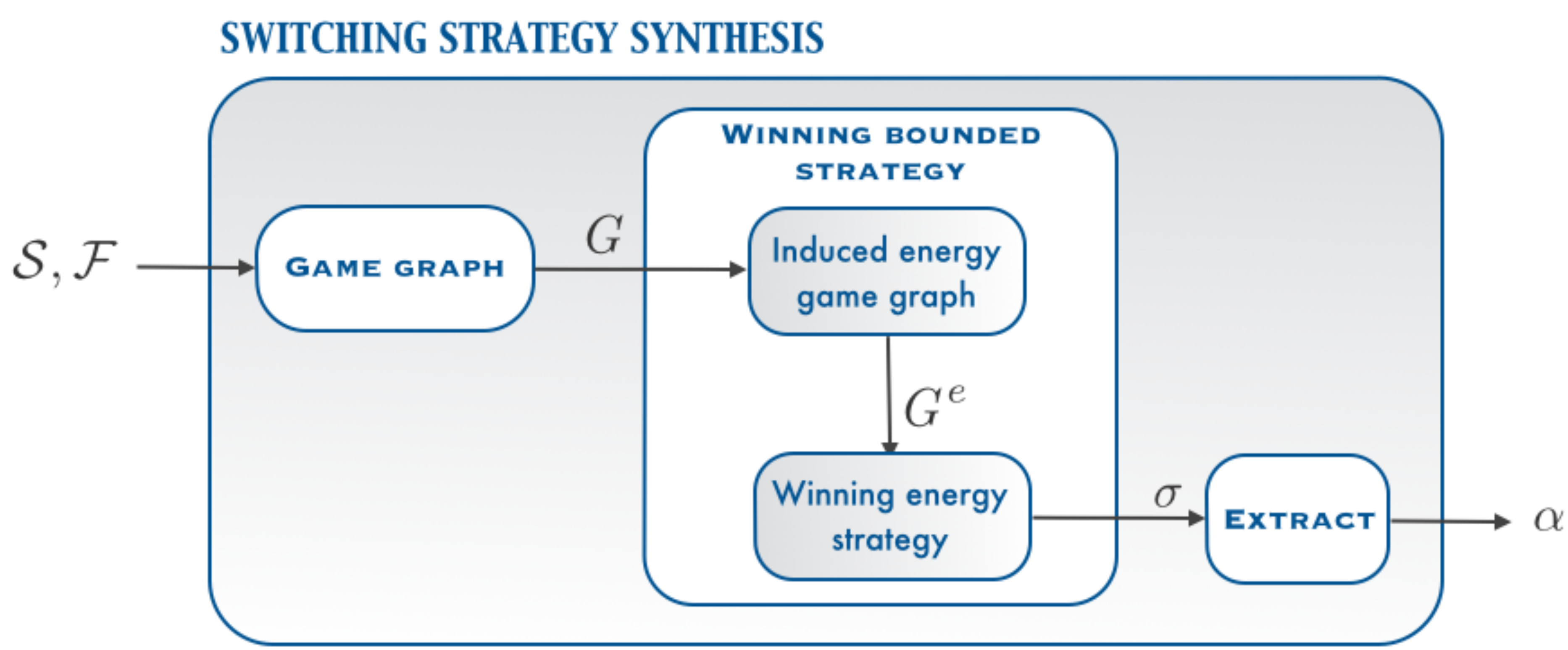
A system is Lyapunov stable with respect to 0 if for every $\varepsilon > 0$ there exists $\delta > 0$ such that every execution x starting from $B_\delta(0)$ implies x $\in B_\varepsilon(0)$.





Overview

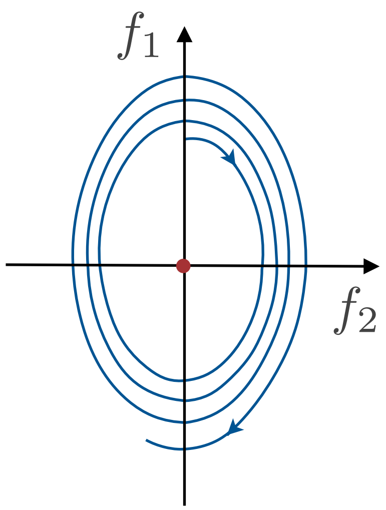
- ☐ Abstract a game graph G from a family of dynamics $\mathcal{S}$ and a set of valid facets $\mathcal{F}$.
- ☐ Induce an energy game graph G^e from G.
- ☐ Compute an energy winning strategy σ from the game graph G^e.
- ☐ Extract a stabilizing switching strategy α from σ.



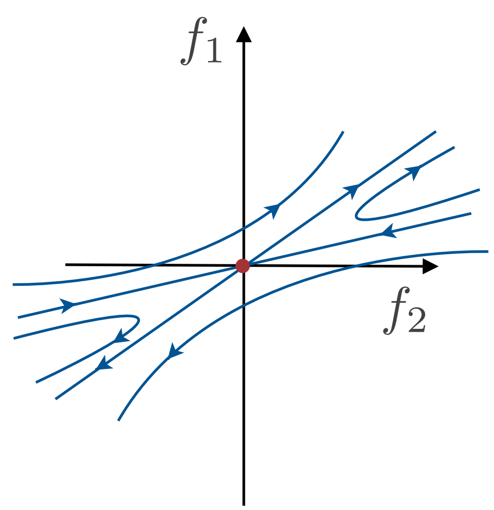
Abstract Game Graph Construction

Quantitative predicate abstraction

$$\mathcal{S} = (\{1,2\}, \{A_1, A_2\})$$

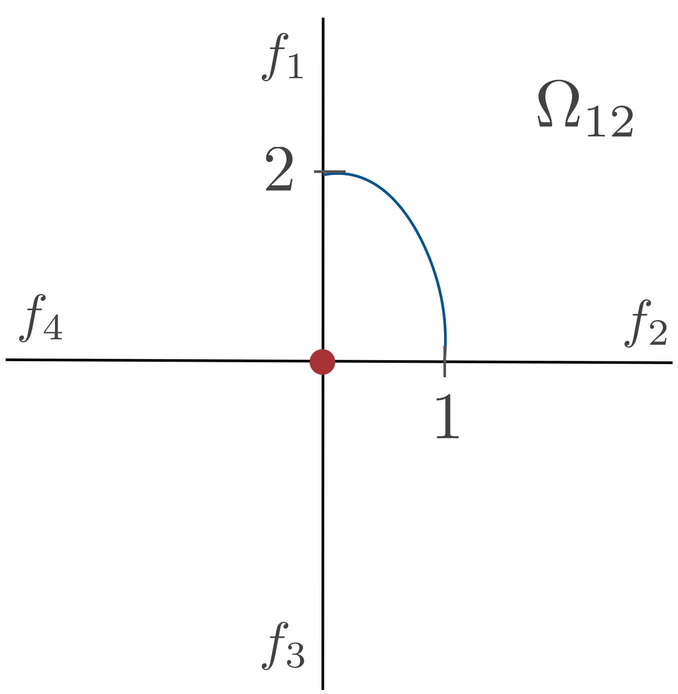


$\dot{x} = A_1 x$

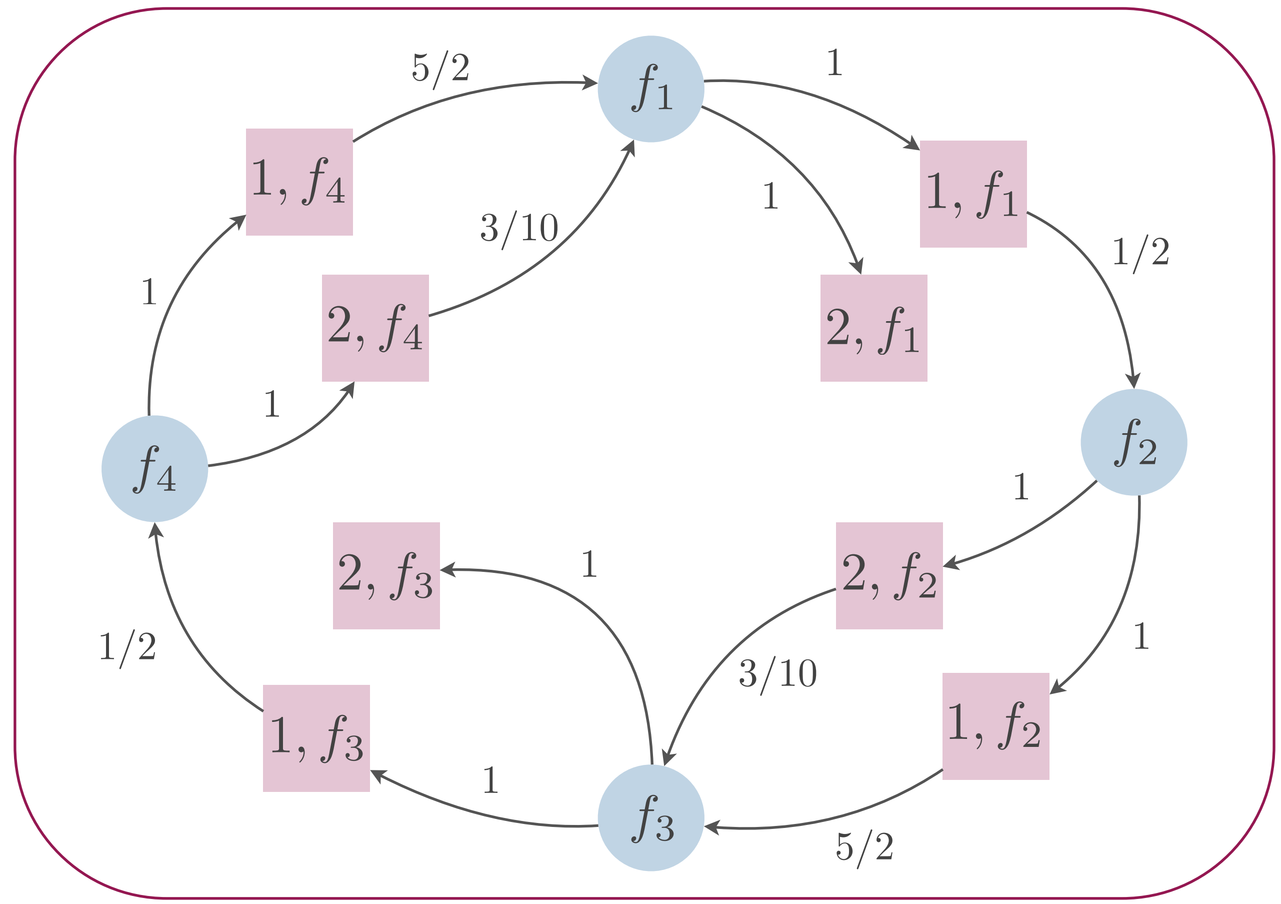


$\dot{x} = A_2 x$

$$\mathcal{F} = \{f_1, f_2, f_3, f_4\}$$



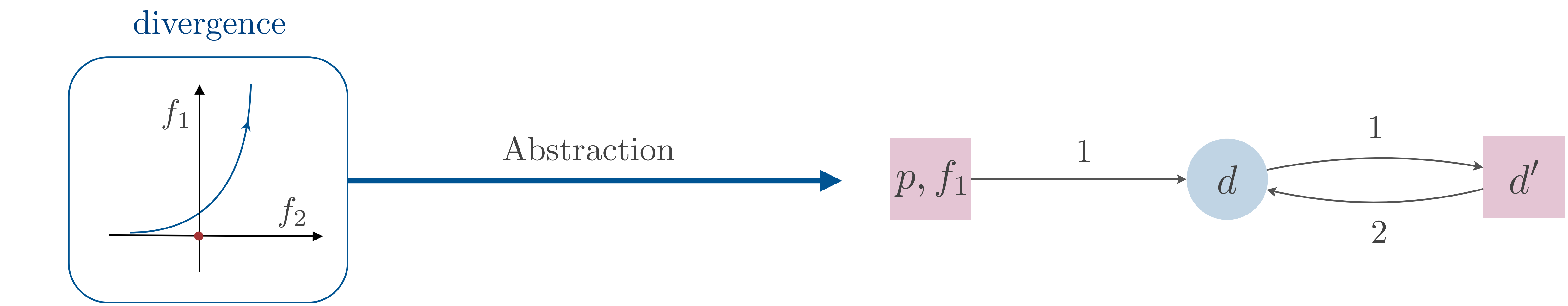
Quantitative predicate abstraction

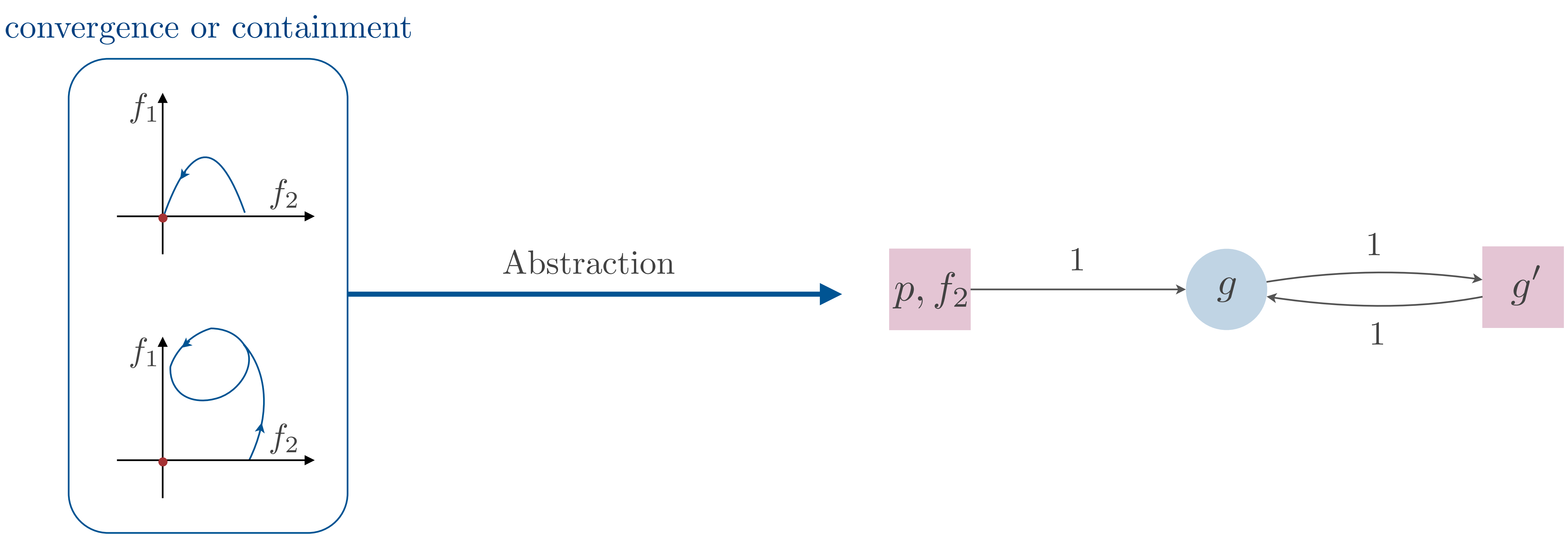


$$W((p, f_i), f_j) = \sup\{\frac{||x_j||}{||x_i||} : x_i \in f_i, x_j \in f_j, x_i \xrightarrow[\Omega_{ij}]{p} x_j\}$$

Ω_{ij} common region of f_i and f_j

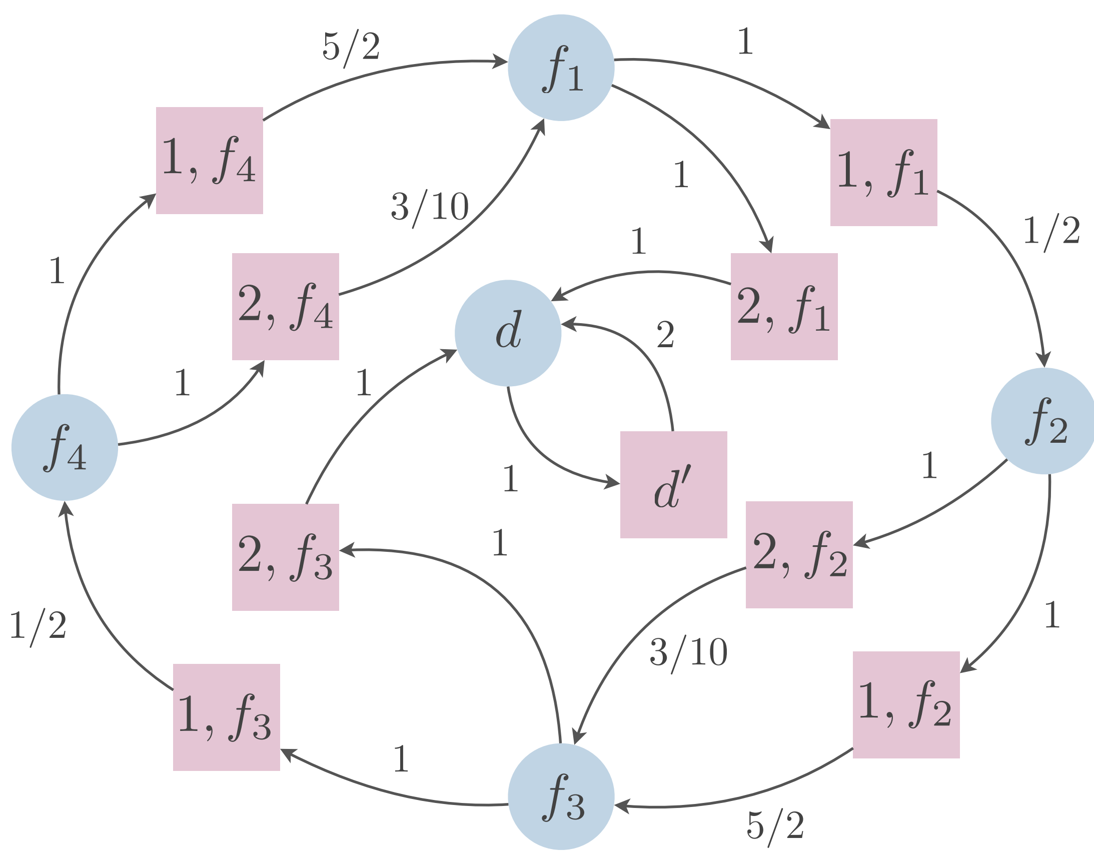
Auxiliary cycles





Strategy Synthesis

Game graph

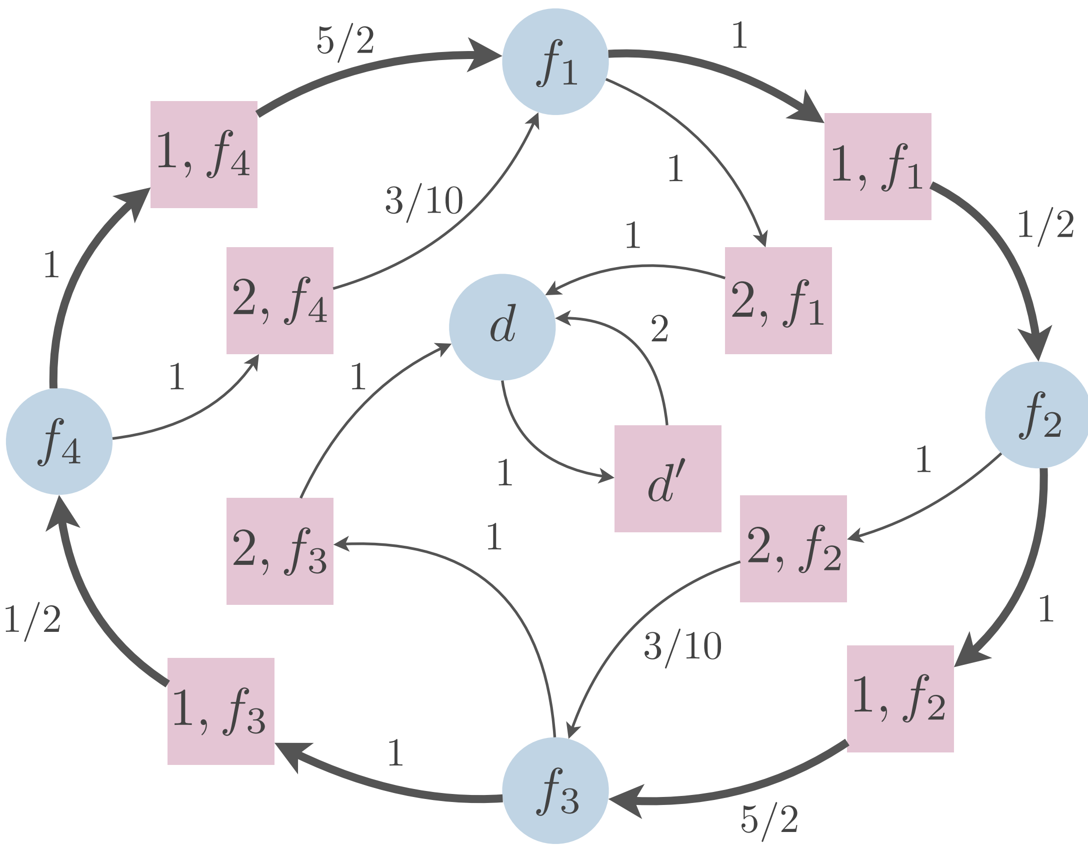


Game graph is a weighted graph G=(V,E,W)

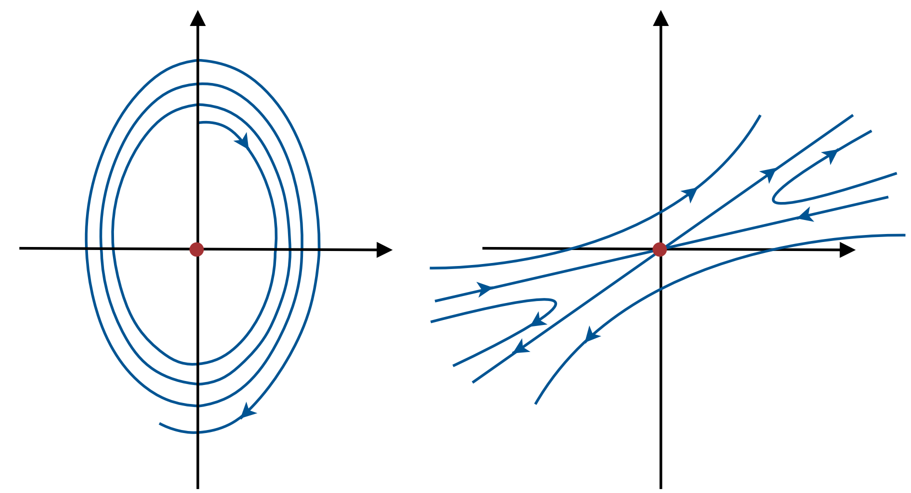
- ☐ $V = V_0 \cup V_1$
- ☐ $V_0 \cap V_1 = \emptyset$
- ☐ $E \subseteq (V_0 \times V_1) \cup (V_1 \times V_0)$
- ☐ $W : E \to \mathbb{Q}$
- ☐ Every node has a succesor

A strategy is a function $\sigma : V^*V_0 \to V_1$, where V^* is the set of finite sequences over V with zero or more elements.

Strategy Example



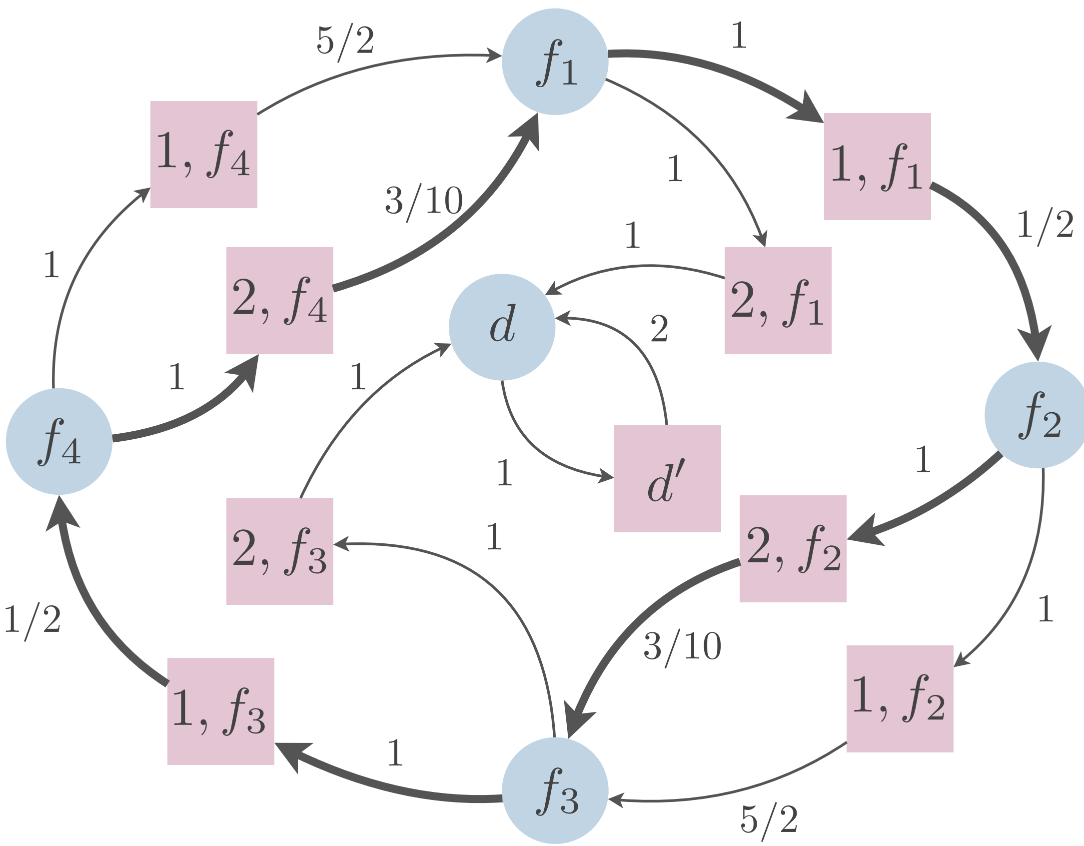
$\mathcal{S} = (\{1, 2\}, \{A_1, A_2\})$



$\dot{x} = A_1 x$ $\dot{x} = A_2 x$

Weight of the cycle is $1/2 \cdot 5/2 \cdot 1/2 \cdot 5/2 > 1$

Strategy Example



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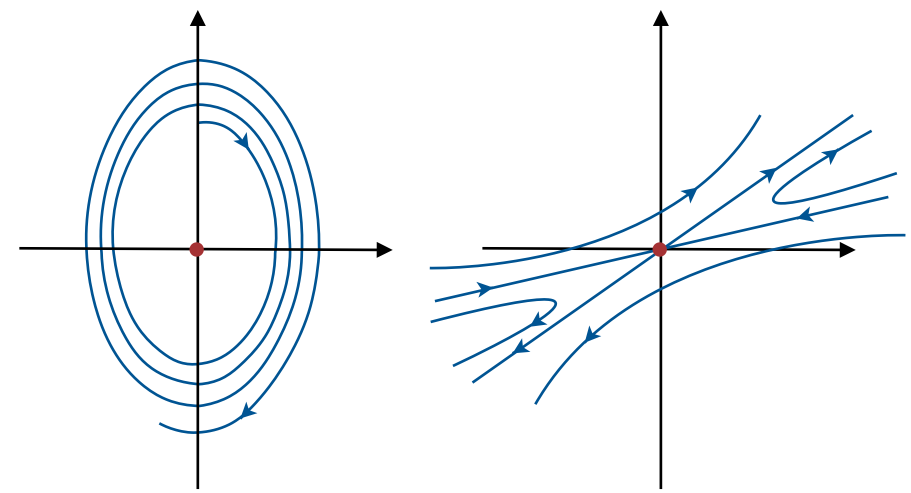

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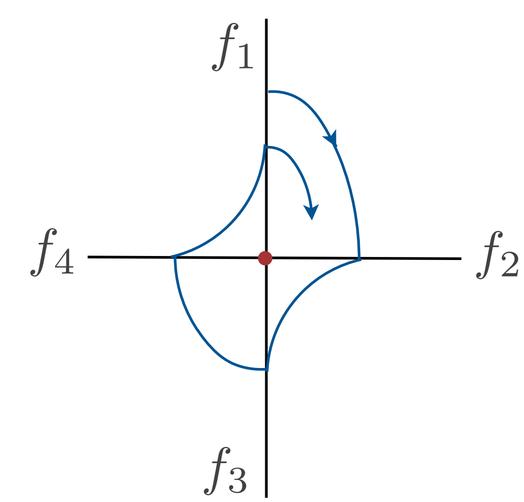


Weight of the cycle is $1/2 \cdot 3/10 \cdot 1/2 \cdot 3/10 < 1$

No cycles with weight greater than 1 implies stability.

Soundness of abstraction

A strategy σ is a winning bounded strategy if there exists $M \in \mathbb{Z}$ such that for every play τ determined by σ, $\mathrm{W}(\tau) \leqslant \mathrm{M}$.

Theorem - stabilizable switching strategy

A winning bounded strategy for the game graph $\mathrm{G}(\mathcal{S}, \mathcal{F})$, induces a strategy which solves the stabilization problem for the system $\mathcal{S}$ and the facets $\mathcal{F}$.

Energy game

A strategy σ is a winning energy strategy if there exists C$\in \mathbb{N}$ such that for every play $\tau = v_1 v_2 \ldots$ determined by σ, C$+ \sum_{i=1}^{j}$W$(v_i, v_{i+1}) \geqslant 0$.

Theorem - energy strategy [Brim et al. FMSD'11]

Given a game graph (V,E,W) where W: E $\rightarrow \mathbb{Z}$, if there exists a winning energy strategy, then there exists a memoryless winning energy strategy. Further, there is an algorithm which returns the memoryless winning energy strategy.

Energy game

Modification of G($\mathcal{S}, \mathcal{F}$) to an energy game graph

- ☐ Reduce multiplicative game graph to addition game graph.
- ☐ Weights are required to be integers.
- ☐ Winning energy strategy provides plays lower bounded by a value.

Bounded game graph

G = (V,E,W)

$$W(e) = \frac{a_e}{b_e}$$

$\text{LCM}_G :=$ least common multiple $\{b_e : e \in E\}$

→

Energy game graph

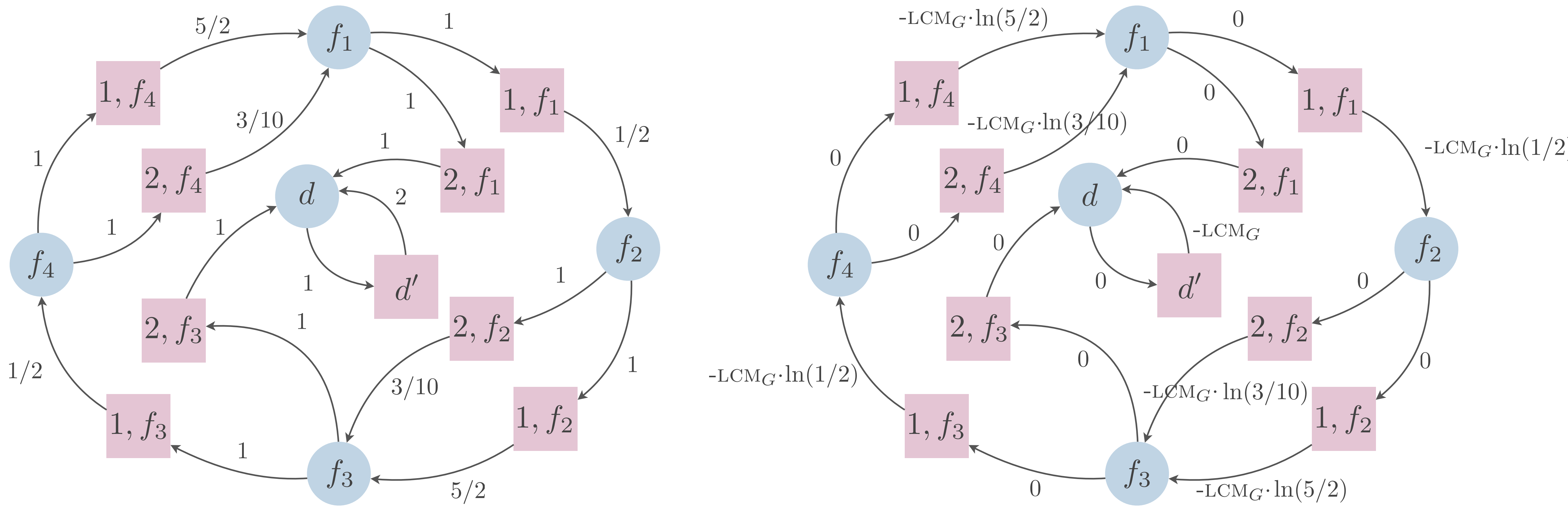
G^e = (V,E,W^e)

W^e = - $\text{LCM}_G \cdot$ W

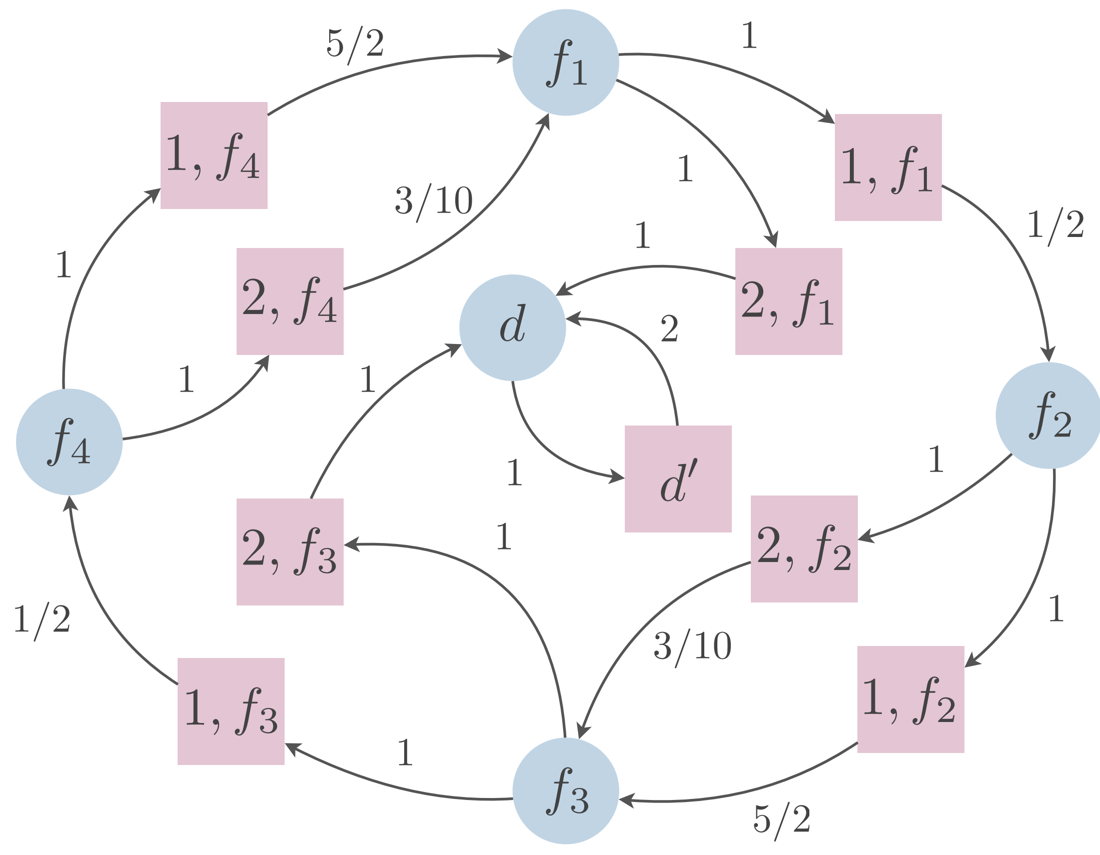
Theorem - bounded strategy

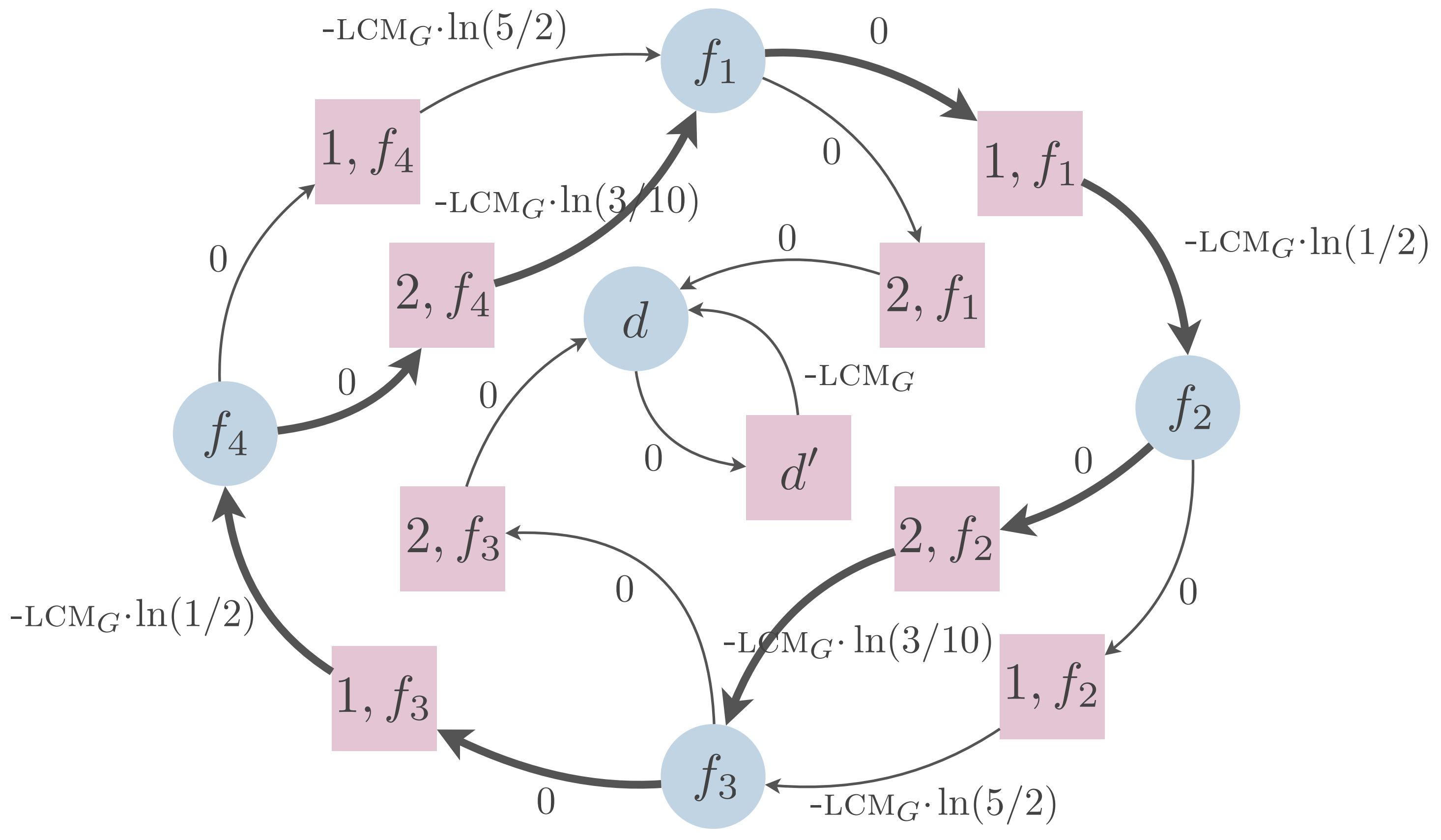
σ is a winning energy strategy for G^e if and only if
σ is a winning bounded strategy for G.

Reduction to energy game



Reduction to energy game





Winning energy strategy

$\sigma : V_0 \to V_1$

$f_1 \mapsto (1, f_1)$

$f_2 \mapsto (2, f_2)$

$f_3 \mapsto (1, f_3)$

$f_4 \mapsto (2, f_4)$

Conclusion

- An abstraction technique and game based approach for synthesizing a switching logic for stabilization.
- Our approach can be combined with temporal logic properties to obtain stable controllers which satisfy the temporal logic formulas.

Thank you