An Algorithmic Approach to Global Asymptotic Stability Verification of Hybrid Systems

Miriam García Soto and Pavithra Prabhakar

IMDEA Software Institute and Kansas State University

Cyber-physical systems (CPS)

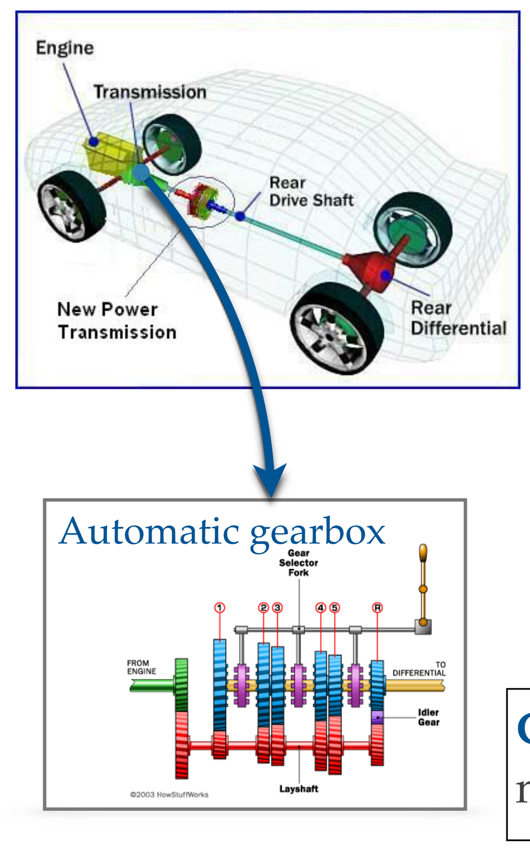
- CPS integrate control, computation and communication
- CPS are safety critical systems
- Need strong guarantees of correctness

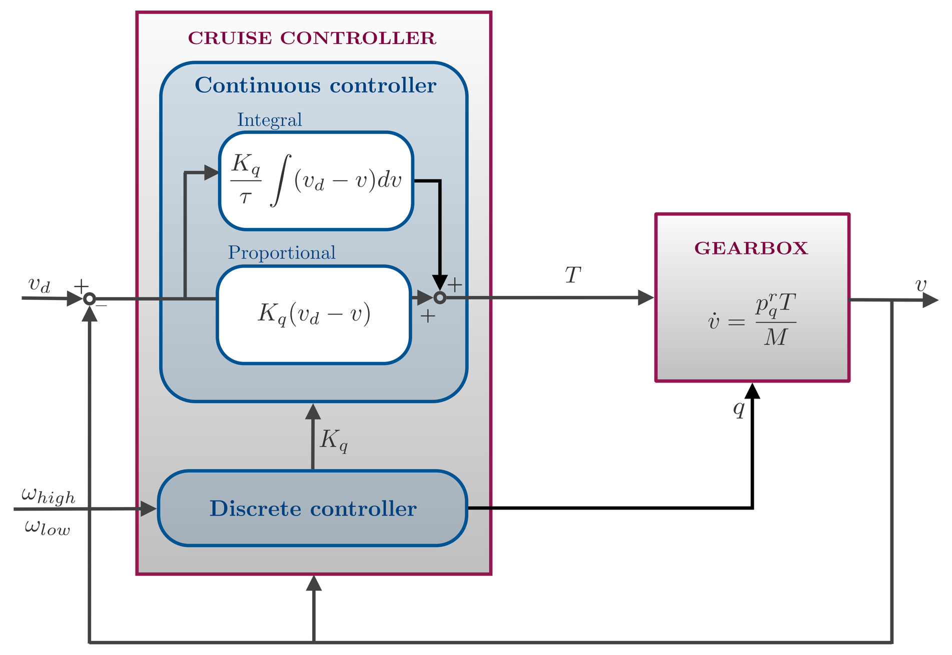
Healthcare: Pacemakers

Transportation: Cars, Aerospace systems

Energy: Smart grids

Cruise control & automatic gearbox





Goal: Drive the vehicle velocity to a desired velocity and maintain it in the presence of perturbations

Stability

- Stability is a fundamental property in control system design
- It captures the property that small perturbations to the input of a system results in only small deviations in the behaviour of the system

Classical notions of stability

- Lyapunov Stability (LS)
- Asymptotic Stability (AS)
- Global Asymptotic Stability (GAS)
- Region Stability (RS)

Global asymptotic stability ensures that the system converges to the equilibrium point starting from any state of the system

Global asymptotic stability verification

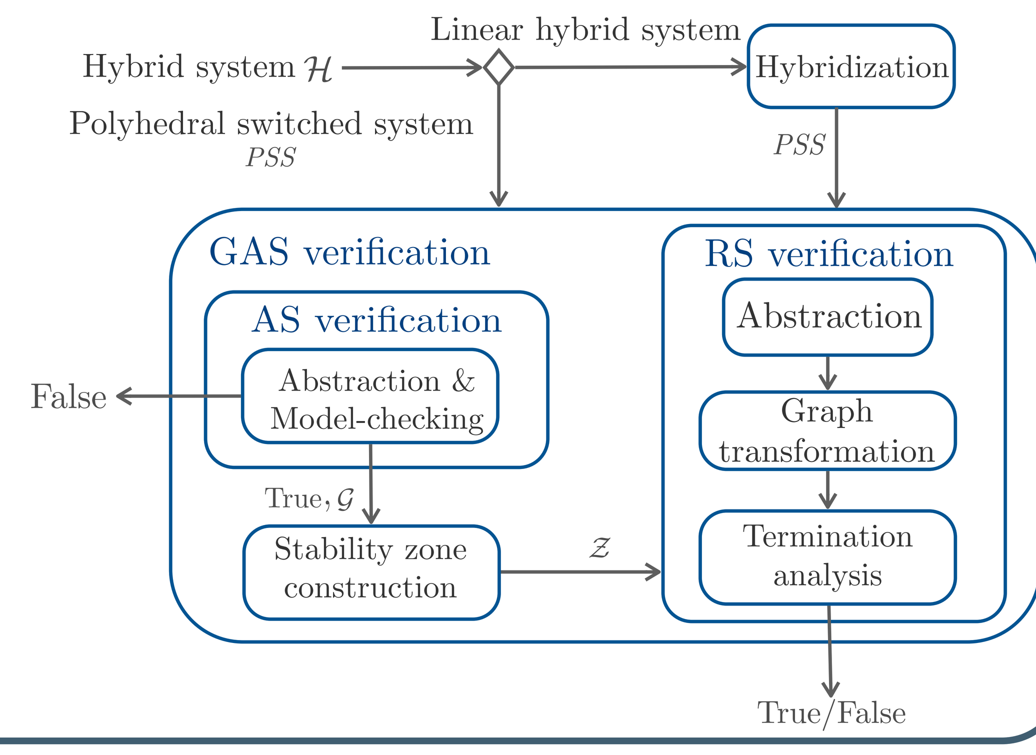
Main idea: Reduce the GAS verification problem to an AS verification problem and an RS verification problem

GAS verification steps

Step 1. Check asymptotic stability

Step 2. Construct a stability zone $\mathcal{Z}$

Step 3. Check region stability with respect to $\mathcal{Z}$



Asymptotic stability verification

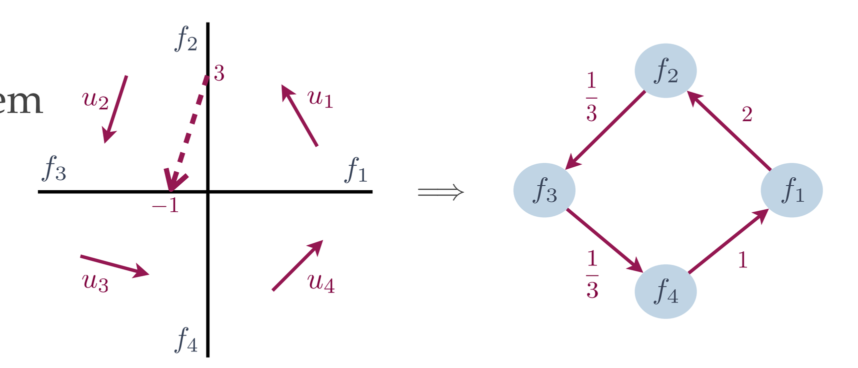
Lyapunov stable - A system is LS with respect to 0 if for every $\varepsilon > 0$ there exists $\delta > 0$ such that for every execution σ starting from $B_\delta(0)$, $\sigma \in B_\varepsilon(0)$.

Asymptotically stable - A system is AS with respect to 0 if it is Lyapunov stable and there exists a value $\zeta > 0$ such that every execution σ starting from $B_\zeta(0)$ converges to 0.

Algorithmic approach

Reduce the problem to graph analysis problem

- Quantitative predicate abstraction of a hybrid system $\mathcal{H}$ into a abstract weighted graph $\mathcal{A}(\mathcal{H}, \mathcal{F})$
- Analyse $\mathcal{A}(\mathcal{H}, \mathcal{F})$ for absence of cycles with weight greater than 1.



Concrete system $\mathcal{H}$

Facets $\mathcal{F} = \{f_1, f_2, f_3, f_4\}$

Abstract system $\mathcal{A}(\mathcal{H}, \mathcal{F})$

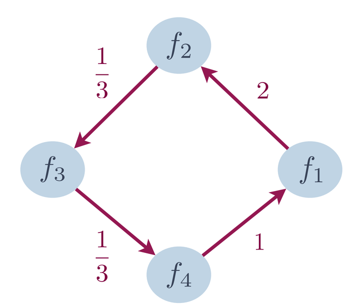
$W(\pi) = 2 \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot 1 = \frac{2}{9} < 1$

Stability zone computation

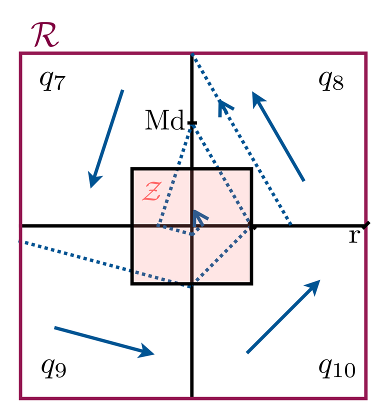
$\mathcal{Z} \subseteq \mathcal{R}$ is a **stability zone** with respect to $\mathcal{R}$ if every execution starting in $\mathcal{Z}$ will remain in $\mathcal{R}$

Construction steps

- Extract the central region from the hybrid system $\mathcal{H}$
- Compute the maximum scaling M associated with the paths in the abstract weighted graph
- Shrink the central region by a factor of M to obtain the stability zone

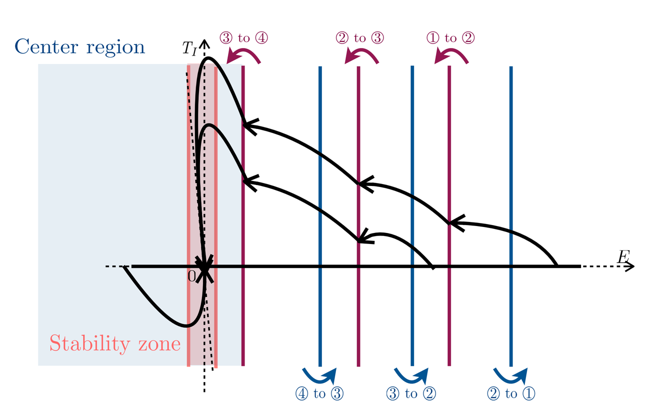


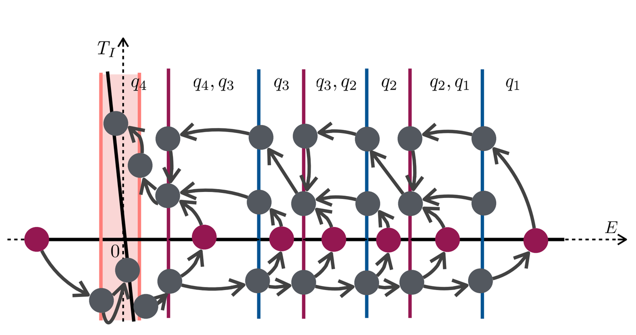
M = 2



Region stability verification

Region stable - A system is RS with respect to R if for every maximal execution σ there exists a value $T \geq 0$ such that σ at time T belongs to R.





Quantitative Predicate Abstraction

Graph Transformation

- Delete nodes in the interior of stability zone
- Delete non-reachable nodes from initial nodes

Termination Analysis

- Existence of an edge with weight $\infty \Rightarrow$ RS False
- Existence of a cycle $\Rightarrow$ RS inconclusive
- Existence of nodes with not outgoing edges different to the nodes on the boundary of the stability zone $\Rightarrow$ RS inconclusive

Current Research

- Extension of the algorithmic stability verification to non-linear systems.
- Compositional analysis for input-output stability verification.
- Synthesis of state based switching control for a family of dynamical systems.

Publications

Counterexample Guided Abstraction Refinement for Stability Analysis, Pavithra Prabhakar and Miriam García Soto, CAV 2016

Hybridization for Stability Analysis of Switched Linear Systems, Pavithra Prabhakar and Miriam García Soto, HSCC 2016

Foundations of Quantitative Predicate Abstraction for Stability Analysis of Hybrid Systems, Pavithra Prabhakar and Miriam García Soto, VMCAI 2015

An algorithmic approach to stability verification of polyhedral switched systems, Pavithra Prabhakar and Miriam García Soto, ACC 2014

Abstraction Based Model-Checking of Stability of Hybrid Systems, Pavithra Prabhakar and Miriam García Soto, CAV 2013

Link: http://software.imdea.org/projects/averist/

Miriam García Soto (miriam.garcia@imdea.org) and Pavithra Prabhakar (pprabhakar@ksu.edu)